Introduction to Homotopy Type Theory — WiSe19/20

Part I, Lecture 11 — Ulrik Buchholtz

Before the break we looked at the homotopical circle. Now we pick back up with *Introduction to Homotopy Type Theory*, by looking at homotopy pullbacks (§ 17) and pushouts (§ 18).

Pullback squares

A commutative square

$$\begin{array}{ccc} C & \xrightarrow{p} & B \\ {\scriptstyle q}\downarrow & & \downarrow{\scriptstyle g} \\ A & \xrightarrow[f]{} & X \end{array}$$

with $H : f \circ p \sim g \circ q$ is a *pullback square* if the *cone* (C, p, q, H) over the *cospan* $A \xrightarrow{f} X \xleftarrow{g} B$ is *universal*, meaning that maps $(C' \to C)$ are equivalent to cone structures of C' (over the same cospan.

Any cospan has a *canonical pullback* $\sum_{(x:A)} \sum_{(y:B)} (f\,x = g\,y)$, and the square is a pullback square if and only if the canonical *gap map*, mapping $z : C$ to $(p\,z, q\,z, H\,z)$ in the canonical pullback, is an equivalence.

Exercises

The numbers refer to the exercises in the copy of *Introduction to Homotopy Type Theory* on the course website.

- Ex. 17.1. Conclude: (c) the loop type $\Omega(A, a)$ of a type A pointed at a is an instance of a pullback.
- Ex. 17.2 (Truncation level and the diagonal of a map).
- Ex. 17.3 (Mirror of a pullback square).
- Ex. 17.4 (Descent for the empty type).